1-3/1-5 Exponential and Logarithmic Functions

Learning Objectives:

1. I can use the rules of exponents to simplify an expression.
2. I can use the rules of logarithms to simplify an expression.
3. I can model exponential growth and decay using a function
4. I can evaluate logarithms.
5. I can solve an exponential equation.
6. I can use the change of base formula.

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Exponential Growth and Decay Formulas

Growth/Decay by Rate	Natural Growth
$y = A(1 \pm r)^t$	$y = Ae^{kt}$
A = Initial amount r = growth/decay rate t = time y = Amount after time t	A = Initial amount k = growth/decay constant t = time y = Amount after time t

$e \approx 2.71828182843.....$

e is an irrational number (like π)

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Compound Interest

$$y = A\left(1 + \frac{r}{n}\right)^{nt}$$

A = Initial amount

r = growth/decay rate

n = number of times per year that interest is compounded

t = time (in years)

y = Amount after time t

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Ex1. A 100 gram sample of a radioactive substance decays so that there is a 75 grams left after 10 days

a.) Find the decay constant and write an exponential decay model of the form $y = Ae^{kt}$

b.) Find the amount left after 50 days

c.) Find how many days it will take for there to be 10 grams of the substance left?

d.) Find the length of time for one half-life.

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Logarithms

$$E = \log_b N \quad \rightleftharpoons \quad b^E = N$$

$$3 = \log_2 8 \quad \rightleftharpoons \quad 2^3 = 8$$

A Log is an Exponent!

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Ex2. Evaluate Each logarithm without the use of a calculator

1.) $\log_5 25$

2.) $\log_3 \frac{1}{9}$

3.) $\log_{36} 6$

4.) $\log_{27} 3$

5.) $\log_4 8$

6.) $\log_{16} \frac{1}{8}$

7.) $\log_{13} 1$

8.) $\log_{10} 0$

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Laws of Exponents

1. Product Property $b^m \cdot b^n = b^{m+n}$

2. Quotient Property $\frac{b^m}{b^n} = b^{m-n}$

3. Power Property $(b^m)^n = b^{mn}$

4. $(ab)^m = a^m b^m$ 5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

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Laws of Logs

- Product Property

$$log(AB) = \log A + \log B$$

- Quotient Property

$$log\left(\frac{A}{b}\right) = \log A - \log B$$

- Power Property

$$\log M^k = k \log M$$

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Common vs Natural Logs

Common Log (base 10)

$$\log_{10} x = \log x$$

Natural Log (base e)

$$\log_e x = \ln x$$

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Ex3. Solve each equation

1. $3^x = 7$

$\ln 3^x = \ln 7$

$x \ln 3 = \ln 7$

$x = \frac{\ln 7}{\ln 3} \approx 1.771$

2. $e^x = 271$

$\ln e^x = \ln 271$

$x \ln e = \ln 271$

$x = \ln 271 \approx 5.602$

3. $4e^x + 7 = 178$

$-7 \quad -7$

$\frac{4e^x}{4} = \frac{171}{4}$

$e^x = 42.75$

$x = \ln 42.75$

$x \approx 3.755$

Change of Base Formula

$$\log_b M = x$$

$$b^x = M$$

$$\log(b^x) = \log(M)$$

$$x \log b = \log M$$

$$x = \frac{\log M}{\log b}$$

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Homework

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